

Time turns the universal kaleidoscope

Ryan S. Westafer

December 1, 2008

Abstract

The idea of a harmonious universe is one with which scientists and philosophers have sympathized for centuries. Of course, many such as Tolman[11], have revealed difficulties with original oscillatory models, typically citing divergent entropy or difficulty with singularities. By scientific judgement and recent developments[2, 10, 8], I am compelled to resort to such a model in the description of time. However, in this paper, we will consider the symmetry of both forward and backward time (and other quantities) as nature's harmonious and balanced arrangement to allow something in spite of nothing.

First I clarify the qualities and pervasive generality of a resonant universe. Second, I highlight and show connections among several observed properties of time including: entropic irreversibility, the fluctuation theorem and Loschmidt's paradox, and the formation of matter and its eventual decay in decoherence and the emission of radiation into an expanding space. Finally, I conclude with corroboration, potential validation, suggestions for our improvement, and a bit of reflection.

Introduction

If all the symphonies of the world played different scores simultaneously, we would likely perceive a cacophonous roar of noise. However, in the locale of each group, the harmony and beauty could be discerned and appreciated. Perhaps it is so with the universe: in some neighborhoods the fluctuations of energy coincide in harmony and in others, and perhaps on the whole, the interference of fluctuations produces a texture so complex that we are unable to distinguish it from nothingness. This is a modern concept of vacuum, one in which energy fluctuations are allowed if the universal energy is conserved.

In this work, I often introduce metaphors to guide our understanding of the dynamics of time and energy in the context of experience and reality. This process of generalization helps us confirm the inherent fundamental truth in the various physical interpretations. My goal is to provide physical descriptions with which we can all sympathize- both scientifically and experientially.

Approach

To examine the fundamental nature of time, our task is to distill the complexities of physics to the more general results¹. The aim of this work is foundational- in essence, we desire to project a wide cross-section of physics onto mathematics in order to illuminate only the most essential and broad solutions. In some ways this process goes against the gradient of modern inquiry, but it should at least offer a more robust result.

Briefly consider the emission of a photon. When and where will it go? In answer to this question, our study will rely upon a transactional interpretation (TI) of quantum physics[2, 5]. To make a superficial statement: every source must have a sink and *vice versa*. So it should be with time and all other parameters of energy.

The Kaleidoscope Analogy

I hope the reader is familiar with kaleidoscopes. If not, I recommend to the reader: look at some pictures online, or build a kaleidoscope- this way the full dynamics are apparent.

Let us assume a universal kaleidoscope which could be used as a closed system. Maybe instead of shards of glass or a multitude of dyed immiscible fluid droplets, the color wheel contains matter and antimatter, dark energy, stars, and even visible blackbody radiation. Then, we give the wheel an impulsive twirl. First everything spins in beautiful synchrony, and then, as dissipative turbulence begins to dominate: small vortices and other interesting arrangements appear, and some bubbles collide with others, colors mix, and maybe sounds are heard. Here we have a beautiful analogy to the unitary evolution of energy over phase space.

Fundamental Action: Rotation

Our viewpoint in looking through this *universal* kaleidoscope is that time is our parameter of change. Truly harmonic (or periodic) occurrences are timeless through this view. This is why we use them to measure time. If time evolves harmony to non-periodic behavior, then we must choose something essentially independent to measure the entropic/non-periodic phenomena. Whether it is the decay of Cesium atoms in a precision instrument, or one's own heartbeat in a dark isolated room, we measure time (and therefore entropy), in discrete periodic epochs. A quantum mechanical result supports this notion, in which we observe unitary transformations (generalized rotations) to act on the state. Just as the photon carries an angular momentum \hbar , Planck's "constant," the evolution of all quantities in an oscillating universe should be through rotation according to a fundamental action of rotation.

Balance seems to be a profound and basic law. Perhaps this may be broader than conservation of energy or any other law. Things may seem out of balance in an instant... but in the next, we trust nature will restore balance. Even when the underlying mechanisms

¹This approach is further advantageous because it limits the breadth of mathematical knowledge required for comprehension.

are not known, *i.e.* when we declare randomness due to our ignorance, we trust that the lucky gambler will eventually lose, or that the particularly fascinating pattern in the kaleidoscope will return in similar form. The notion of conservation *in time* is both intuitive and pervasive. But where does one demarcate a universal boundary? Though we may consider only a local system with good accuracy: how big is the biggest system? If an observer turns the kaleidoscope, what provides the opposing force? His other hand? Perhaps we need to look through the kaleidoscope while standing in front of a universal mirror (of all action). Such questions we will address in this work, in relation to all variables we perceive in the kaleidoscope... not only time. As we shall see, they are all interrelated.

Physical Nature of Time

In answering the question, “how do we arrive at the kaleidoscope analogy,” we must first relate physical observations to this abstract notion of time, which we hope to elucidate.

We would like to obtain a simple and intuitive understanding which agrees with physics. However, let us note the intricacies of physical interactions and the exceptional cases we eventually seem to find in all corners of physics. Is there one immutable law? We will take as an example the conservation of energy. Others seem generalizable to this law- even the conservation of mass, which was once a law itself, now corresponds to the conservation of energy by Einstein’s famous relation. We will refer to these “laws” as we examine human observations regarding time.

Change

Here I will make a simple note before we proceed: we quantify the passage of time through periodic phenomena (which are nearly timeless), and we record history through deviations from the strictly periodic.

Constants

Will a signal ever have a truly constant value? Is any system infinitely stable? At present we have no observations allowing us to answer “yes” to these questions. In fact, we cannot. It would require an infinite time to make such an observation. What’s more, we notice that any measurement must extract some energy from the system under observation, in accordance with the information obtained. This is troublesome for the idea of constants because such a quantity must be unobserved- both by us or any other system- to remain unaltered. Thus, we cannot know truly universal constants. Likewise, I will argue that absolute constants cannot exist. They are only approximately constant and in a relative sense, as in “constant with respect to something else.”

To manage the complexity of physical phenomena, we search for manageable relationships and trends. For simplicity we typically seek constants and linear functional relationships.

With an energy minimization approach, this is possible. If we let the frequency of a signal be exceedingly low² several things happen:

- We lose the fine detail of the function
- In our neighborhood of observation we see a linear function
- A phase shift still produces any linear slope available to the function according to its frequency.³

This behavior leads the way for a cosmological constant or other treatments of universe inflation in which linearity is sufficient, though we must remember the universal (co)sinusoidal temporal fluctuation of energy.

Eigenfunctions (Normal Modes)

In solving classic partial differential equations and boundary value problems, we use the separation of variables approach in which time dependence and spatial variations are separated. One function simply modulates the other. Relating this to the ever ubiquitous “wave function” we see that a time dependence is again just a multiplied factor $T(t) = e^{i\omega t}$ times a function of other variables $X(\mathbf{x}) = e^{i\mathbf{k}\cdot\mathbf{x}}$. When we write them together we have the typical plane wave we recognize from optics and acoustics: $e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}$. In quantum physics we refer to this multiplicative time function as the time propagator $U(t)$ [9]. We notice that the propagator is a function periodic in t , or a sum of such functions: a Fourier series⁴.

Conservation of Energy

Conservation of energy is typically written as a sum of terms which equate to a constant (or zero?). We might admit some ignorance here. Can we think of instances in which the conservation of energy appears violated? Now this is usually a complication of how we choose to bound a given system.

What can we say about the photon? If a photon passes a point in space, we know the electric and magnetic fields - which we measure to be in-phase in free space - give rise to an energy flux in the propagation direction. This is given by Poynting’s vector. This represents the capacity of the photon to do work on a charge located at the point of observation. However, this electric potential appears and vanishes with twice the frequency ω corresponding to the energy of the photon.

As a simple example of sinusoidal power transmission, we visualize an incandescent bulb (a simple resistor) powered by 60 Hertz mains voltage in the United States⁵. Power is

²Relative, of course to our capacity for measurement or “sampling”

³Steeper changes correspond to greater derivatives, and under the Fourier transform, this corresponds to greater frequencies (by a multiplicative factor in the functional definition).

⁴Or, we allow any other compatible expansion performed by a unitary operator

⁵In our simple analogy, we assume negligible distances and transmission line treatment is unnecessary.

conveyed during both the positive and negative phases, each occurring once per cycle. So this implies the bulb would flash at 120 Hertz if only the filament cooled quickly enough. Application of the square-law for power and use of a trigonometric identity reveal this fact.

Still, the bulb is flashing. Where is the energy when the voltage across the bulb is zero? At such instants, no work is done at the bulb, yet we know (after counting a few prior cycles) that the potential will soon return. The answer is that the energy is in the generator and, further still, in the steam supplied to the turbine. Even if the steam to the generator turbine is interrupted, the rotating machinery carries some angular momentum and will continue to “freewheel” a while, only gradually decreasing the output. This guarantees the lamp will light again a few times, but with less intensity each time. Thus, even in this ideal and simple circuit without capacitors or inductors, there is so-called “reactive power” stored (in many possible forms) in the direction of the power source. This is the key point of the whole analogy: *when the time-harmonic energy has a zero real (dissipative) value, the imaginary component (anti-dissipative) peaks*. Thus there exists a time-harmonic balance of energy source and energy sink. Recall our transactional interpretation: every source must have a sink, no matter the scale.

If energy is really conserved, then what about entropy? Do we have a hidden garbage chute for the universe? If a process is to be irreversible then we expect some opposing energy exists which prevents reversal. If a ball rolls to the bottom of a parabola in the presence of a gravitational potential, we know what constitutes this stable equilibrium- its rest energy. But how did the ball come to rest? Without friction, this problem would have only a time harmonic solution, so this reveals dissipation and entropy increase. We will see that the trap door of the universe lies in the mirror image; it’s all contained there in the “kaleidoscope.”

We see Schrödinger’s equation as one of balance.

$$i\hbar \frac{\partial \Psi(\mathbf{r}, t)}{\partial t} = \hat{H} \Psi(\mathbf{r}, t)$$

where, typically

$$\Psi = e^{i(\omega t - \mathbf{r})}$$

The wave function (Euler and Fourier’s complex exponential) is simply an energy balance coefficient which shifts energy on each side. This achieves a time-periodic balance indeed, and depending upon our choice of basis, the complicated structure of interactions (the tangle of which physicists strive to unravel at the Large Hadron Collider) is provided in the Hamiltonian. Over a complex Hilbert basis, a universe of infinite structure - and harmonious balance - exists⁶. Now, pressing further, we see that one side of the equation is completely “imaginary.” I have searched in the literature for explanations, but one appears absent. However, after reading this work, I hope the reader will appreciate the operation the Hamiltonian, H , constitutes. This operator is the unitary operator which transfers energy from

⁶We can parameterize the phase space in many different ways, but the result is still harmonic with respect to any one independent parameter of the phase

the real basis to the imaginary. Thus, energy is conserved, whether we have previously been able to figure out where it goes, or not. In fact, the concept of entropy is neatly presented in this equation. We see that changes of energy- that is *all changes*- are described by action of the Hamiltonian consistent with an increase of “imaginary” energy. We see the rate of change of Ψ is with respect to “time” (the only “external” parameter we see). Energy is lost to a domain orthogonal to our real existence- this agrees with entropy: information to disinformation and cold to hot.

If Ψ is an energy balance coefficient, then the reader may ask why we always speak of the “probability amplitude.” The present interpretation according to this work, is that the likelihood of a state can be determined by an interference of its energy balance coefficient (wave function) with the rest of the universe. This is neatly performed in the integral for expectation value. All states under this model are inherently fluctuating, so it appeals to intuition that when we weigh one fluctuating energy against another which we cannot observe, we gain a measure of likelihood for observation.

Direction and Reversal

Here we consider entropy and the “arrow of time.” We perceive time to have a direction due to the irreversibility of processes. This is what makes one moment distinct from the previous and the next.

Dissipation in phase space (Boltzmann-Shannon) seems to be universally preferred. That is, if we compute a phase space volume for an initial state then any subsequent state exhibits at least an infinitesimally larger volume. In simple terminology: the energy of a state at a later time is spread over a larger basis. For instance, if heat is ultimately produced in the course of dissipation (and this is always the case) then the energy spreads in the the space. In quantum terminology, this is decoherence.

Is this just a result due to our phase space neighborhood (reference frame) as living organisms? Perhaps this begs invocation of the anthropic principle. Remember that measurements are bound by the response of the measuring device. Our perception is limited by the construction of our bodies and the measurement tools we use.

In a fluctuating universe, time evolution should be no exception. Though we observe time’s arrow as irreversible, *i.e.* monotonic and identified with the increase of entropy, we may consider this an observational necessity. By the anthropic principle and observed causality, we must exist in the inflationary entropic universe. Perhaps the *universal* fluctuation is hidden, spread out over state space, or else it is simply perceived linear within our ability to measure.

Just as negative numbers arise in our attempt to produce zero under a linear function, the “imaginary” numbers arise in attempt to produce zeros of a quadratic equation. So it seems for ages we have desired to to solve for balance, and we reject the trivial solutions, because they do not yield the structure we see. Without reaching so far as topology and the fine texture of mathematical physics, I think it is time we investigate the full ramifications of the complex exponential as a basis function, and the essential complementarity it provides.

Many have taken time to be a fourth dimensional parameter of the periodicity, and this is a good approach. However, the problem is with the formulation by Einstein in 1912, $u = ict$ [4]. Time a “lumped” fourth parameter in addition to space? If we assume a complex u , we only have half the picture. For everything to hold, we may take c complex. This provides the necessary avenue for dissipation while allowing the conjugate phase balance necessary in an oscillating universe. Time reversal is essential, but we cannot observe it as anything other than the future. As the future (reverse time) interferes with the past, we find ourselves on a surface of the present. John G. Cramer even wrote a paper on the “plane of the present,” so he and I seem to concur in the application of the transactional interpretation[3].

CPT Symmetry

The transactional interpretation acknowledges the source and sink matching required to destroy a “particle” into what I will recognize as vacuum energy. The collapse of the wavefunction in a transactional way removes time dependence because of phase cancellation of conjugate states. This is the important point of CPT symmetry which leverages both charge conjugation and spatial inversion (parity) to preserve causality.

Next we will visualize the zero Dirichlet boundary condition often taught in undergraduate level differential equations courses, or in physics in conjunction with the d’Alembert solution to the wave equation. This is the one where solutions comprise both backward and forward traveling waves. There is a special treatment of this boundary condition which works well and is conceptually helpful for the student. By the principle of superposition, as a wave approaches the boundary, it is appropriate to achieve the zero value at the boundary by passing a wave of opposite polarity through the boundary in the opposite direction. In this way the boundary is like a mirror, and we have applied what is called the “method of images.” This is frequently used in electromagnetics and ought be applied in the case of such cosmological boundary conditions. Hence, in a more general form, we have CPT symmetry—all realized through time reversal.

Singularities, Relativity, and Time Dilation

In cases of spherical symmetry, or symmetry centers, we note that the three spatial variable parameters necessarily approach zero along a phase trajectory toward the spatial origin. To maintain energy in each infinitesimal spatial step, time must scale relative to the step. In brief, this accounts for time dilation and Lorentz contraction.

Black holes have been proposed as suitable universal boundaries. Then black holes are the thermal sinks into which all radiation travels. So then, where are the white holes? This theory indicates that they are inside us and all around us— the luminous matter of the universe. If we think of the blackbody radiation of all matter, this begins to appeal more strongly to reason. For instance, we know blackbody radiation agrees with the “disinformation” requirement of Hawking’s radiation. From one singularity to another, energy flows unidirectionally... but anti-energy flows in reverse. As time advances, we experience dissipation as the conjugate universe experiences the opposite. I suppose the event horizon could be realized in the

vacuum. Out of the vacuum come Cramer’s “offer waves” and into it go “confirmation waves”- photons. Perhaps it is no coincidence the stars break down matter into radiation and even at the shell of each atom, see electrons chaotically transitioning. More complex energy packets may of course be emitted; their cross-sections correspond to the various particles and this process could similarly be called “Hawking’s nuclear decay.”

Mass and Stationarity

How does inertia arise in a harmonic universe? I’ll give an explanation according to our model; we must of course assume a wave theory of matter. In such a theory, a stationary distribution of energy is produced by the interference of “traveling” waves. Thus, the concept of a standing wave applies here just as in the classical examples of introductory physics. When two waves pass one another at the same velocity (opposite wave vectors \mathbf{k} and $-\mathbf{k}$ and at the same frequency), an energy pattern is formed which does not translate with respect to the axis of propagation. This pattern still oscillates with time. The momentum of the waves is now manifest only in position⁷. The reader may recall holographic images are recorded by the process of interfering waves. Some recent theories of the universe imply a high-dimensional holographic model. This is quite similar to the present model.

So, how do we plausibly stop light to produce matter? We simply allow a complex speed of light, as is so frequently used in practice, and as we shall soon recall. If the real velocity of light is reduced, then its counterpart, the “imaginary,” must be increased. Einstein’s famous $E = mc^2$ relation assumes a constant speed of light, and it is only this constancy which we will revise, and fortunately it will *not* invalidate the relationship, in that we may use cc^* instead of c^2 . Of course, the magnitude of this complex quantity does not change, so in that sense, Einstein’s relation is preserved and his quantity is constant. Justification follows.

Though not often used in cosmology, it is well known in optics and electromagnetics that the speed of light may be treated as a complex quantity. This was first used with much success in the scattering theory work of Kramers and Kronig in 1927 and 1926, respectively[6]. Now the general relationship connecting the real and imaginary parts for causal (analytic) signals is used extensively in practice. It even holds for nonlinear optical phenomena. A reference is provided for more information about the applicability of these relations; in short, causality and finite energy are required [7].

The general result of the Kramers-Kronig relations demonstrates that a change in wave velocity with frequency (dispersion) necessarily implies a change in attenuation. Intuitively, this is true: when some frequency components of a Gaussian wave packet are slowed (relative to the rest), destructive interference reduces the amplitude of the packet.

If light passes “through” matter, it slows in that inertial frame⁸. We as observers must be somewhat co-moving with respect to the matter we are observing, or we would be unable to measure it. Ultimately the wave velocity may approach zero; its energy then reinforces the resonance and stationarity of the matter, and of course, for energy balance, its wavelength

⁷And frequency, but we assumed constant frequency in this analysis, which limits the energy phase space

⁸As is well known in the theory of refraction

must approach zero. Here we have the "crunch" of wavelengths into a singularity.

From a scattering theory perspective, this is maximum absorption; the wave maximally interferes with the matter. This of course depends upon frequency content of the wave packet; it must be constructed to "overlap" the appropriate interaction cross-section for the matter under observation. Other destructive influxes of energy will perturb the system, *i.e.* the waves will "beat" together, and this will produce a phase shift such as translation in space.

While time ultimately appears periodic, the periodicity is degraded as the universe expands and spatial wavelengths are stretched. The standing energy of matter is lost to longer wavelengths, and we see blackbody radiation and thermal decay as spatial distributions of energy assume larger wavelengths, along radial lines away from matter; the singular centers of spherical symmetry.

Uncertainty Principle

In quantum mechanics, Heisenberg's uncertainty principle appears in conjunction with the various operators corresponding to physical observables such as position, momentum, energy, and time. While I will not repeat the basics here, an important result follows from commutation of the quantum mechanical operators. If two physical observables can be known precisely *and simultaneously*, then the corresponding operators (*e.g.* A and B) commute ($[A, B] = 0$).

Our fundamental inquiry is with the fact that the canonical conjugate operators (*e.g.* position and momentum) do not commute. When applying two conjugate operators in a different order, we are left with an imaginary factor of the energy (or probability amplitude, as some would say). The factor is typically something like $i\hbar$. This seems to imply that our dissipative measurement swaps energy and anti-energy in an increment of Planck's "constant." Here again, the time forward and time reverse interpretation may have been fruitful.

Corroboration and Validation

In view of all the observations, generalizations, and connections we have made, it is necessary to frame-in the generalities and attempt to apply them.

Universally Fundamental

It was an intersection of Cramer's "transactional interpretation" (TI) [2] and Lynds's "Finite Universe" [8] which reverberated in my mind. Both concepts rely upon basis functions which admit both time forward and time reverse solutions. As one reviewer of Lynds's paper remarked at FQXi.org, the universe must go "bang, crunch, bang"- what a great way to

phrase it⁹.

Why would we search for unbalanced singular solutions of the cosmos when a Fourier series solution would fit just as well? I argue we must not preclude the periodic solution which rests upon such a suitable and ubiquitous basis. After all, Fourier series solutions¹⁰ are appropriate for the solutions of all differential equations which come to mind¹¹.

Unfortunately, it may be surprising (or the reader may recall) that strictly periodic functions do not actually satisfy the absolute integrability requirement of our powerful Fourier transform, *i.e.* Fourier transforms of such functions require the special treatment of generalized functions[1]. A very generalized point of this detail could be that nature may very nearly repeat itself, but not exactly. This is one reason physicists apply Gaussian wave packets with such success- they meet the requirements for Fourier transformation and are observed in practice.

If there is a universal fundamental time period, maybe it is connected to entropy... the largest scale time dependence of which we are aware. Thomas Gold believed the cosmological arrow (expansion) is connected with the thermodynamic arrow (entropy). We too can show such connection using the Green's function solution to the heat equation. However, for brevity, I will just give the result. As we substitute larger times into the function $G(x, t)$, the spatial variance of the fundamental Gaussian-form solution increases. Thus, simply, as time advances, the distribution spreads. This corresponds also to a spatial scaling. The universe expands, heat spreads, and "things" cool. The conservation of energy is fundamentally constrained by space in addition to time. This is why we relate the two by the speed of light, an apt choice for a constant *universal* rate of expansion.

In a possible validation of the present work, Loschmidt's paradox may be resolved. This paradox is the contradiction of time symmetry and entropy. This is the contradiction apparent in the Fluctuation Theorem. The paradox arises because of an improper assumption: time asymmetry was used to predict time asymmetry. If we consider that, in an oscillating universe, completely reversible thermodynamic classical collisions are allowed, they fit the time symmetry inherent in this oscillating universe model. Yes, entropy may fluctuate "the other way." However, we simply note that the reverse trajectories for particles are undetectable, because they occur in phase conjugate form and are simultaneous with respect to the dissipation we observe. This is truly a mirror universe... not only mirrored in time, but in everything: all energies.

So, while this work does not reveal the actual geometry of the universal kaleidoscope and it does not predict all of the beautiful textures we will see, it seems to piece together an appropriate mechanism.

⁹Although, after much study it seems the "bang" and "crunch" we speak of occur simultaneously by reflection.

¹⁰Or relatives such as the Hankel and Hilbert transforms

¹¹Even for nonlinear partial differential equations which may admit discontinuous shocks as solutions, the Fourier transform converges in the mean

Infinite Structure

A simple confirmation of this approach would reveal infinite structure under unitary transformations. As computational mathematicians search for repeating digits in π , this could be a futile endeavor (or at least a waste of energy). Perhaps the corresponding physical result can be rigorously proved. Regardless, I find it interesting that the fundamental mathematics should rely upon incomputable infinite series. Here again lies a sense of duality. Though we aim to reduce physics to mathematics, still, the very constants themselves become impossible to measure precisely. Maybe this is an analogous uncertainty principle in mathematics, and perhaps in this way we may know we have reduced our mathematics sufficiently, because the physically observed duality appears in numbers we can never know exactly. There is much to rigorously prove here, only time will tell when and how scientists and mathematicians will devote energy in such pursuits.

Further Work

Of particular interest for continued study is the congruence of the various generalities with the intricate interactions of particle physics. In treating only a general phase space without regard to “charge,” “color,” or any other taxonomy of energy state beyond space-time, much work remains in the details. As stated in the validation section, the only immediately apparent test of the claims is to demonstrate infinitesimal structure- and this might be impossible. Furthermore, if it were true, endless energy would be required to blast nucleons apart and process all the scattering data on computers. Still quantitative results are needed to demonstrate agreement for the strengths of the interactions, *etc.*

Additional work also lies in critique. This article did not address uncountably many possible contingencies. Remember, much of modern physics pursues one singularity or the other (zero or infinity). As when gazing into a fractal image, at some scale we are content to appreciate its beauty- and our inherent limitations- and press no further in those directions. I recognize many weak statements in the foregoing, and I hope for some lively criticism.

Reflection

To conclude our discussion let us recall famous words which have been oft recited throughout history:

History repeats itself. -Thucydides

Should we challenge this age-old adage? I suggest we may to a small degree. If history repeated itself exactly then it would not appear to be history at all; we would lose track of time in boring re-runs as one can find on television. Indeed, nature provides a balance in which very complicated and rich interactions are possible in an irreversible way according to our frame of reference. So we have much to inspect and enjoy in this epoch- this twirl of the kaleidoscope.

References

- [1] R.N. Bracewell. *The Fourier Transform and Its Applications*, pages 10, 94. McGraw Hill, 3 edition, 1999.
- [2] J.G. Cramer. The Transactional Interpretation of Quantum-Mechanics. *Reviews of Modern Physics*, 58(3):647–687, July 1986.
- [3] John G. Cramer. The plane of the present and the new transactional paradigm of time, 2005.
- [4] A. Einstein. Manuscript on the special theory of relativity, 1912.
- [5] R. E. Kastner. Why the afshar experiment does not refute complementarity, 2005.
- [6] R. de L. Kronig. On the theory of dispersion of x-rays. *J. Opt. Soc. Am.*, 12:547–557, 1926.
- [7] V. Lucarini, J.J. Saarinen, K.-E. Peiponen, and E.M. Vartiainen. *Kramers-Kronig Relations in Optical Materials Research*. Springer, 2005.
- [8] P. Lynds. On a finite universe with no beginning or end, 2006.
- [9] R. Shankar. *Principles of Quantum Mechanics*, page 146. Springer, 2 edition, 1994.
- [10] L. Susskind. The World as a hologram. *J. Math. Phys.*, 36:6377–6396, 1995.
- [11] R. Tolman. *Relativity, Thermodynamics, and Cosmology*. Dover (1987), 1934.